Since the most common financial payoffs are convex, as for example plain vanilla call and put, the idea of convex comparison of martingale measures arises quite naturally.

We say that $Q_1$ is dominated by $Q_2$ in the convex order if it gives lower prices to each convex payoffs; this is equivalent to say that the distribution of the underlying under $Q_1$ is dominated in the convex order by its distribution under $Q_2$. Since all martingale measures have the same mean (that is, the forward price), the elementary necessary condition for convex comparison is always automatically satisfied.

Convex ordering is underlying many papers related to the comparison of option prices under different models, such as ([7], [8], [3], [4]) in continuous time and ([10], [6]) in discrete time, and is also underlying much of the literature on option pricing bounds, that are provided by martingale measures that are extremal in the convex order.

In a previous work ([2]) we showed that in a finite, discrete time model the Esscher and the (local) minimal entropy martingale measure are always comparable in the convex order, and which one is dominating depends on the sign of the risk premium of the underlying; in the typical case of a positive risk premium the Esscher measure is dominating and hence it gives higher prices to each convex payoff.

In this work we deal with the problem of convex comparison of minimal divergence martingale measures, that are defined as the martingale measures minimizing an $f$-divergence in the sense of [5] and [9].

As it is well known, this minimization problem arises naturally as the dual of a utility maximization problem and hence these prices have an economical interpretation as utility prices.

A particular subclass is constituted by the power divergences, that include also as special cases the relative entropy and the inverse relative entropy, as well as many common statistical distances such as the Hellinger or the Chi-square distances.

Limiting ourselves to the case of finite-discrete time markets, we are able to prove that minimal power divergence measures are always comparable in the convex order and hence produce ordered call and put prices. Similar results have been obtained in the context of stochastic volatility models by [8] (see also the references therein).

We give also sufficient conditions, based on the divergences or dually on the utilities, that guarantee convex ordering of the corresponding measures; the induced ordering of option prices is investigated both from a theoretical and a numerical point of view.
REFERENCES


