Optimal Investment Strategies for Insurance Companies in the Presence of Standardised Capital Requirements

Katharina Fischer* and Sebastian Schlütter

Abstract

The standard formula of the Solvency II framework employs an approximate value-at-risk approach to define risk-based capital requirements. The parameterization of the standard formula determines how much additional capital insurers need in order to back investments in risky assets. This paper investigates how the standard formula’s stock risk calibration influences the equity position and investment strategy of a shareholder-value-maximising insurance company. Intuitively, a higher stock risk parameter should reduce the insurer’s risky investments as well as his insolvency risk. However, by considering the insurer’s equity level as an endogenous variable, we identify situations in which a stricter stock risk calibration leads to a significant reduction of stock investments, but leaves the actual solvency level virtually unaffected, since the insurer also lowers his equity capital position. While previous articles only deal with the statistical accuracy of the standard formula’s calibration, our results shed light on the incentives resulting from different calibrations.

Keywords: solvency regulation, capital requirements, asset allocation, insurer default risk

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1 Introduction

Solvency II, the future framework for insurance regulation in the European Union, is a significant move away from rules-based regulation and toward principle-based regulation. In particular, the new standard will not impose any direct limitations on an insurer’s asset allocation and explicitly accords “freedom of investment” in Article (Art.) 133.¹ Instead of static rules, future solvency regulation will build on a three-pillar framework. Pillar I contains quantitative risk-based capital requirements, which are supposed to ensure that the insurer’s annual ruin probability is below 0.5%. Capital requirements can be determined either by using an internal risk model developed by the insurer and approved by the regulator, or by means of a standard formula defined in the Solvency II framework. The standard formula determines the capital requirement based on various risk modules and sub-modules. For instance, insurers are required to hold more capital when investing heavily in risky assets. The magnitude of this effect depends on the calibration parameter for stock risk in the standard formula. In addition to capital requirements, Solvency II will provide qualitative requirements for governance and risk management in Pillar II, and disclosure and transparency requirements in Pillar III.

Despite the promised “freedom of investment” principle, some scientists and practitioners have expressed fear that Solvency II could imply lower stock positions of insurance companies, which in turn could create financing problems for the real industry and possibly endanger insurance returns. The standard formula is said to demand excessive capital for stock holdings, as opposed to government bonds. Existing articles have pointed out weaknesses and arbitrariness in the statistical calibration and claim that the standard formula does not account for long-term effects such as mean-reversion of stock returns.²

To investigate the question whether the standard formula requires excessive capital from a theoretical perspective, one has to distinguish between two possible cases. On the one hand, it is possible that insurers have sufficient self-interest to hold capital and that the Solvency II capital requirements or regulation in general is not necessary. As shown by Rees et al. (1999), insurers will raise sufficient equity funds to ensure perfect solvency if policyholders can observe the insurer’s default risk level and regulators do not impose restrictions on their asset allocation. In this case, insurers will invest a portion of their assets into risky stocks in order to attain efficient portfolio allocation. On the other hand, there could be an information

¹ Directive 2009/138/EC.
asymmetry or a commitment problem between the insurer and policyholders, implying that insurers will not willingly hold high equity levels and thus that regulatory capital requirements are necessary.  

In this article, we focus on the case where binding capital requirements are necessary and are determined using a standard formula similar to that contained in Solvency II. We analyse the insurer’s optimal combination of equity capital and risky stock investments. To this end, we take into account that the shareholder value of a (stock) insurance company can be interpreted as a call option on the future equity capital of the firm. Option pricing theory implies that the insurer can raise shareholder value by increasing the volatility of its future portfolio, e.g., by investing in risky stocks, and by maintaining a low asset-liability ratio, e.g., by holding less equity capital. As the standard formula restricts the admissible combinations of equity capital and risky stock investments, it defines the boundary of the shareholder-value maximization and determines the insurer’s preferred equity level as well as its asset allocation.

Our results show that the calibration of the standard formula can have a strong influence on the insurer’s optimal stock investments and the default risk level: when confronted with a lax stock risk calibration of the standard formula, the insurer will decide on a high share of stock investments and hold additional equity capital according to the standard formula. As the standard formula is lax, however, the required capital add-on is too low and the insurer attains a clearly higher default risk level. When confronted with a strict stock risk calibration, the insurer will avoid any risky investments and keep its equity capitalization to a minimum.

We also demonstrate that a change in the stock risk parameter can have ambiguous consequences depending on its current level. If the stock risk parameter is currently relatively strict, a small change in the parameter could influence the investment strategy but will not have a significant effect on insurer default risk. However, if the parameter is below a certain threshold, even a slight adjustment can have significant consequences for the insurer’s investment strategy resulting in a considerable increase in insolvency risk. We discuss these results in light of different assumptions as to the correlation between assets and liabilities.

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On the one hand, our results add some credence to the industry’s fear of Solvency II by verifying the coherence between the strictness of the standard model calibration and the insurer’s optimal stock investments. On the other hand, our results point out that regulators should be very careful with an alleviation of the model risk calibration so as to avoid an excessive increase of insolvency risk.

The paper is organised as follows. Section 2 contains an overview of the relevant literature. Section 3 explains the modeling framework. Section 4 investigates the impact of the standard model calibration on optimal investment strategies and insurer default risk. Section 5 discusses the results and provides policy implications. Section 6 concludes.

2 Literature Overview

The argument that risk-based solvency regulation with a one-year time horizon could have undesirable side effects on insurers’ asset allocation is raised by van Bragt et al. (2010) as well as by Bec and Gollier (2009). Van Bragt et al. (2010) analyse how capital requirements based on a simplified standard formula influence a life insurer’s risk-and-return profile. The authors investigate the capital requirements resulting from different investment policies and demonstrate that the model induces a reduction in short-term risk that can drive down the long-term expected returns. Bec and Gollier (2009) investigate horizon effects for French data. Even though stocks have a higher risk in the short run, they have the advantage of being slightly mean-reverting. Bond and bill returns, however, are mean-averting. Thus, given a longer investment horizon, stocks will be less risky than they appear in a short-run consideration.5

Both of the above-mentioned articles argue that the one-year value-at-risk might make insurers focus too much on their one-year risk profile and thus deter them from stock investments, which could be superior in a long-run analysis. The theoretical model of Filipovic et al. (2011) shows that capital requirements can reduce the insurer’s potential to invest riskily. The authors argue that the feasibility of risky investments causes a commitment problem between the insurer and its shareholders, and that capital requirements may help mitigate this problem. However, the article assumes that stock risk can be measured directly under the value-at-risk, and hence does not investigate the influence of the calibration of an

5 This argument goes back to Campbell and Viceira (2002, 2005), who provide empirical evidence for the United States.
approximate risk measure as the standard formula. Gatzert and Schmeiser (2008) compare capital requirements resulting from a value-at-risk-based framework to those from a tail-value-at-risk approach. They find that under both measures, insurers’ who fulfill the respective requirements can still have different levels of default risk measured in terms of the default put option value. Using a simulation study with a multi-period time horizon, Wiehenkamp (2010) analyses how a life insurance company will adjust its investment strategy to different regulatory regimes. He shows that both the standard model and the internal model under Solvency II reduce the insurer’s optimal risky asset allocation and thus lower insolvency risk. The author also points out that the parameters of the standard formula (i.e., shocks and correlation coefficients, see section 3.1) could fail to restrict the default risk to the desired 99.5 percent confidence level. Focusing on stock risk, Mittnik (2011) identifies several weaknesses in the calibration of the Solvency II standard model. He points out that annualising daily returns to calculate the one-year value-at-risk makes the value-at-risk estimates, which form the basis for the standard formula, highly unstable and arbitrary. Also, correlations might be implied even if the data are independent, whereas truly existing dependencies suggested in the historical data might be lost. With regard to the specifications of the fifth Quantitative Impact Study (QIS5), Hampel and Pfeifer (2011) identify a bias in the quantile estimation regarding premium and reserve risk. This bias results from the QIS5 assumption of an expected loss ratio of 1. The authors suggest a formula for calculating an undertaking-specific standard deviation that can be used in the Solvency Capital Requirement calculation to help overcome the bias.

In total, the literature identifies several problems with measuring risk based on a simplified standard formula. However, to the best of our knowledge, there is no analysis focusing on the consequences of changes in the model parameterization. By employing option pricing theory as the measure for an insurer’s shareholder value, we investigate the consequences of different standard model parameterizations for the capital and investment strategy of a value-maximizing insurance company.

3. Model Framework
We consider an insurance company in a one-period setting. At time 0, the insurer receives premiums in the amount \( \Pi \). Shareholders endow the company with equity in the amount of \( K \) and, hence, the insurer’s initial assets are given by \( A_0 = \Pi + K \). The percentage share of assets invested in a risky stock \( M \) is denoted by \( \sigma \). The insurer decides on an optimal
combination of \((K, \alpha)\) under the objective of maximizing shareholder value (SHV). The SHV is determined under the risk-neutral measure \(Q\). The regulatory standard formula acts as a restriction in the optimization problem.

### 3.1 The Regulatory Standard Formula

We incorporate regulatory capital requirements using a standard approach similar to the standard formula in the Solvency II framework. The standard formula determines the overall Solvency Capital Requirement (SCR) as an approximation of the company-wide value-at-risk. It is calculated as the sum of the basic solvency capital requirement \((BSCR)\), the capital required to cover operational risk, and an adjustment position accounting for the fact that technical provisions and deferred taxes can absorb losses. The \(BSCR\) consists of risk modules for non-life underwriting risk, life underwriting risk, health underwriting risk, market risk, and counterparty default risk, as well as defined sub-modules of these risks.

The Solvency II standard formula is a modular approach. In a first step, individual SCR\(_s\) for sub-modules are calculated. The SCR for stock risk and also for some types of liability risk are evaluated based on scenarios and calculated using the delta-net asset value approach. The insurer’s net asset value is defined as the amount by which assets exceed liabilities. The delta-net asset value is the difference between the current net asset value as of the balance sheet date and the net asset value that would result upon the occurrence of a specified regulatory scenario. The SCR for the respective sub-module is given by

\[
    SCR_k = (assets - liabilities) - (assets_{\text{shock}} - liabilities_{\text{shock}})
\]  

(1).

In a second step, the individual SCRs for sub-modules are aggregated to individual SCRs for risk modules and those, in turn, are aggregated to the company level. The aggregation is accomplished by an approximate correlation approach, which gives exact results, e.g., under a normal distribution assumption:

\[
    BSCR = \sqrt{\sum_{i,j} \text{corr}_{i,j} \cdot SCR_i \cdot SCR_j}
\]  

(2).

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8 Cf. CEIOPS (2010, p. 10), EC (2010, SCR.1.5., SCR.1.6., SCR.7.16., SCR.7.28., SCR.7.50., SCR.7.64., SCR.7.75., SCR.7.82., SCR.8.31) and Kochanski (2010, p. 39).
with $i$ and $j$ denoting risk modules and $corr_{i,j}$ being the correlation between two risk modules as defined in the directive.\textsuperscript{10}

In our model framework, we incorporate a simplification of the Solvency II standard formula by focusing on stock risk. To keep the model tractable, we use a simplified sub-module for the liability risk and ignore operational risk and deferred taxes.\textsuperscript{11} To calculate the delta-net asset value, we assume that stock risk does not influence liabilities and vice versa.

Consequently, using Equation 1, we determine the $SCR$ in the stock risk module in proportion to the insurer’s stock risk position

$$SCR_{stock} (\alpha) = shock_{stock} \ast \alpha \ast (\Pi + K)$$

(3),

with $shock_{stock}$ denoting the regulatory assumed shock. The $SCR$ for liability-related risk $SCR_{liab}$ is determined as

$$SCR_{liab} = shock_{liab} \ast L_{0}$$

(4),

with $shock_{liab}$ being the shock the regulator assumes to mirror liability-related risk. Both capital requirements are merged into the overall $SCR$, according to Equation 2 and thus

$$SCR(\alpha) = \sqrt{(SCR_{stock} (\alpha))^2 + 2 \cdot corr \cdot SCR_{stock} (\alpha) \cdot SCR_{liab} + (SCR_{liab})^2}.$$

In line with the Solvency II directive, we set $corr = 0.25$.\textsuperscript{12} Thus, the regulator demands that the insurer holds equity at least in the amount of $K^{reg} (\alpha) = SCR(\alpha)$.

3.2 Modeling Asset and Liability Risks

We model the insurer’s asset and liability risks via geometric Brownian motions. Under the risk-neutral measure $Q$, the insurer’s asset process is defined by

$$dA_t = r_A dt + \sigma_A W_t^Q,$$
with \( r_f \) the risk-free interest rate, \( \sigma_A = \alpha \cdot \sigma_M \), \( \sigma_M \) the instantaneous diffusion parameter of the risky asset \( M \), and \( W_{M,j}^Q = W_{M,j}^Q \) denoting a Brownian motion under \( Q \). The liability process is given by
\[
dL = r_f L_t \, dt + \sigma_L L_t \, dW_{L,j}^Q,
\]
with \( \sigma_L \) the instantaneous diffusion parameter and \( W_{L,j}^Q \) a Brownian motion under \( Q \). The Brownian motions are correlated according to
\[
dW_{A,j}^Q \, dW_{L,j}^Q = \rho \, dt,
\]
with \( \rho \) being the correlation between assets and liabilities.

### 3.3 The Insurer’s Target Function

We next formulate the insurer’s target function. Taking into account that shareholders are protected by limited liability, \( SHV \) is given by
\[
SHV = \exp(-r_f) \mathbb{E}_Q [\max\{A_t - L_t; 0\}] - K
\]
\[
= \exp(-r_f) \mathbb{E}_Q [A_t - L_t + \max\{L_t - A_t; 0\}] - K
\]
\[
= A_0 - L_0 + DPO_0 - K
\]
\[
= \Pi - L_0 + DPO_0
\]

\[
(5)
\]
with \( DPO_0 = \exp(-r_f) \mathbb{E}_Q [\max\{L_1 - A_1; 0\}] \).

For the subsequent procedure in this article, it is important to note that \( DPO_0 \), and thus \( SHV \), depend on \( K \) and \( \alpha \). Given that risks evolve according to geometric Brownian motions, we can incorporate this relation using Margrabe’s (1978) formula, thus
\[
DPO = L_0 \times N(z) - A_0 \times N(z - \sigma)
\]
\[
(6)
\]
Here, \( N \) denotes the cumulative standard normal distribution function, while
\[
z = \ln \left( \frac{L_0}{A_0} \right) \times \frac{1}{\sigma} + \frac{\sigma}{2}
\]
and
\[
\sigma = \sqrt{\sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A\sigma_L} .
\]
\( \sigma \) corresponds to the volatility of the insurer’s liability-to-asset ratio \( \left( \frac{L_0}{A_0} \right) \). On the one hand, \( \alpha \) influences \( \sigma \), as \( \sigma_A = \alpha \cdot \sigma_M \). \(^{14}\)

For higher values of \( \alpha \), the asset-liability portfolio becomes more risky, which increases

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as well as $SHV$. On the other hand, $K$ influences the liability-to-asset ratio $\left( \frac{L_0}{A_0} \right)$. A higher value for $K$ implies a lower liability-to-asset ratio $\left( \frac{L_0}{A_0} \right)$, which lowers insolvency risk, $DPO_0$, and thus $SHV$.

We assume that there is no interaction between insurance pricing and the asset allocation and thus fix $\Pi$ as well as $L_0$. Therefore, $DPO_0 = DPO_0(\alpha, K)$. Furthermore, we assume that the insurer has no incentive to exceed the regulatory capital requirement, and thus the capital requirement $K^{reg}(\alpha)$ defines the equity level the insurer will hold corresponding to the chosen risky investment share. The insurer’s optimization problem therefore simplifies to determining an optimal investment strategy.

4 Optimal Investment Strategies

4.1 Model Calibration (Base Scenario)

We next employ a simulation study that allows us to explore the insurer’s optimal investment strategy under different calibrations of the standard formula, and represent their interconnectedness graphically. To this end, we parameterise the model insurer as follows. The time-0 value of insurance liabilities is set to $L_0 = 2500$, so is the premium income, thus, $\Pi = 2500$, suggesting that the expected underwriting return is zero. Under these circumstances, $SHV(\alpha, K) = DPO_0(\alpha, K)$, according to Equation 5. The diffusion parameter of the risky asset process $\sigma_M$, as well as the diffusion parameter of the liability process $\sigma_L$, are set to 0.15 (cf. Yow and Sherris, 2008, pp. 307–309). In the first part of our analysis, we set the correlation coefficient between asset and liability risks $\rho = -0.25$. Later, we consider a sensitivity analysis for this parameter.

In the base scenario, the regulatory standard model is calibrated as follows. We set $shock_{\sigma} = 0.4$, which is consistent with the technical specification of the QIS5 of Solvency
II. Furthermore, we assume $\text{shock}_{\text{lab}} = 0.4$. This reflects that $\sigma_A = \sigma_L$, as in the parameterization of the model insurer. Finally, we set the regulatory correlation coefficient $corr = 0.25$\textsuperscript{16} so as to match the Solvency II parameterization.\textsuperscript{17}

4.2 Influence of the Market Risk Parameter in the Standard Formula

In a first step, we derive the insurer’s shareholder-value-maximizing stock investment in the base scenario. In our model framework, we therefore need to discover whether the insurer is able to achieve a higher $DPO$, and thus a higher $SHV$, by adopting a more risky investment policy, even if this implies holding more equity capital according to the standard formula. The insurer’s optimal strategy can be understood graphically by looking at Figure 1. The solid line in Figure 1 is the capital curve, which corresponds to all those combinations of $\alpha$ and $K^{res}(\alpha)$ that just meet the capital requirements under the assumed parameters.\textsuperscript{18} We see that $K^{res}(\alpha)$ is an increasing function in $\alpha$, meaning that the standard formula requires additional capital for a higher stock risk. In addition to the capital curve, we show several iso-$DPO$ curves in the diagram. These curves depict those combinations of $(\alpha, K)$ that lead to a constant $DPO$ value $DPO(\alpha, K) \equiv \text{const}$.

If the insurer invests completely risk-free $(\alpha = 0\%)$, the standard formula requires an initial equity capital $K^{res}(0\%) = 1000$, resulting in $DPO(0\%, 1000) \equiv 1.91$. The dashed line in Figure 1 depicts the iso-$DPO$ curve that corresponds to $DPO(\alpha, K^{res}(\alpha)) \equiv 1.91$. If the insurer invests all assets in risky stocks $(\alpha = 100\%)$, we have $K^{res}(100\%) = 2421$, resulting in $DPO(100\%, 2421) \equiv 0.52$. The dotted line marks those combinations of $\alpha$ and $K$ that result in $DPO(\alpha, K(\alpha)) \equiv 0.52$. Due to $SHV(\alpha, K) = DPO_y(\alpha, K)$, we have $SHV(0\%, 1000) > SHV(100\%, 2421)$, i.e., the insurer will aim at investing all assets into risk-free securities, resulting in $DPO = 1.91$.

Since the regulator overestimates the asset risk in the standard formula, the adoption of a risk-free asset allocation allows for a relatively strong reduction in required initial equity capital,

\textsuperscript{15} While QIS4 suggested a shock of 32 per cent, QIS5 calculations would have had to be based on a 39 per cent shock, which is considered as the correct parameter for a “normal” economic cycle phase. Due to the financial crisis, the parameter has been adjusted to 30 per cent in QIS5. Cf. EC (2010, e.g., SCR.5.33 and SCR.5.34).

\textsuperscript{16} The different signs of $\rho$ and $corr$ only result from the design of the formulas.

\textsuperscript{17} Cf. Directive 2009/138/EC, Annex IV.

\textsuperscript{18} See Section 3.1.
which in turn leads to the highest $DPO$. Visually, this result occurs as the slope of the capital curve is lower than that of the $DPO$ curves over the entire range of $\alpha = 0$ to $\alpha = 1$.

**Figure 1**  Combinations of equity capital and stock investments in the base scenario with $\text{shock}_s = 0.4$.

![Graph showing combinations of equity capital and stock investments](image)

We now modify the base scenario by changing the stock risk parameter to $\text{shock}_s = 0.2$. The solid line in Figure 2 shows the new capital curve. If the insurer decides on a risk-free asset allocation ($\alpha = 0\%$), he needs to hold equity capital $K(0\%) = 1000$, resulting in $DPO(0\%,1000) = 1.91$. The dotted line depicts those combinations of $\alpha$ and $K(\alpha)$ that lead to $DPO(\alpha, K^{\text{reg}}(\alpha)) = 1.91$. If the insurer invests only in risky stocks ($\alpha = 100\%$), $K(100\%) = 1416$, and $DPO(100\%,1416) = 8.33$. The combinations ($\alpha$, $K$) with $DPO(\alpha, K(\alpha)) = 8.33$ are depicted by the dashed line in Figure 2. Since $DPO(0\%,1000) < DPO(100\%,1416)$, the insurer maximises its $SHV$ by engaging in a high-risk investment strategy. Compared to Figure 1 we see that the slope of the capital curve is now higher than that of the iso-$DPO$ curves. Therefore, the regulatory underestimation of stock risk implies that the insurer can hold risky stocks and continue to face relatively low additional capital requirements.
Figure 2  Combinations of equity capital and stock investments for $\text{shock}_{st} = 0.2$.

In each of the above cases, corner solutions are optimal, i.e., the insurer invests all assets either in risk-free securities or in risky stocks. However, it may also be that an interior solution is optimal. To illustrate such a case, we set $\text{shock}_{st} = 0.3$.

Table 1  $DPO$ resulting from different investment strategies given $\text{shock}_{st} = 0.3$

<table>
<thead>
<tr>
<th>$\alpha^*$</th>
<th>0.75</th>
<th>0.80</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K(\alpha^*)$</td>
<td>1505</td>
<td>1562</td>
<td>1622</td>
<td>1686</td>
<td>1754</td>
<td>1826</td>
</tr>
<tr>
<td>$DPO(\alpha^<em>, K(\alpha^</em>))$</td>
<td>2.73</td>
<td>2.75</td>
<td>2.77</td>
<td>2.76</td>
<td>2.75</td>
<td>2.74</td>
</tr>
</tbody>
</table>

Given that $\text{shock}_{st} = 0.3$, Table 1 shows that the insurer would optimally decide to invest 85 percent of its assets in risky stocks, allowing it to reach $DPO(85\%, 1,622) = 2.77$.

We next generalise the analysis of optimal investment strategy for different values of $\text{shock}_{st}$, and consider the whole interval $\text{shock}_{st} \in [0.2, 0.55]$. Figure 3 depicts the optimal share of risky investments (solid line) as well as the corresponding default risk (dashed line) depending on the regulatory assumed shock for stock risk. Figure 3 demonstrates that
When we combine the influences of the stock risk parameter on investment strategy and insurer default risk, we can distinguish between four different cases. In the first—$shock_R$ smaller than 0.28—a stricter stock risk parameter causes a significant reduction in insurer default risk, while the insurer’s optimal investment strategy is unaffected. Even though stock investments are accompanied by additional capital requirements, the insurer still finds it optimal to hold only risky stocks. In the second case—$shock_R$ between about 0.28 and 0.35—a stricter stock risk parameter is accompanied by a more conservative investment strategy as well as a reduction in default risk. It is notable that within this relatively narrow interval of model parameters, the optimal stock investment decreases rapidly from 100% to 22%. Hence, in this situation, a slight reduction of the stock risk parameter might lead to a significant change in the investment strategy as well as a dramatic increase in insurer default risk. In the third case—$shock_R$ within about 0.35 and 0.37—a stricter parameter leads to clearly less stock investment $\alpha^*$, but has hardly any effect on the safety level. In the fourth case—$shock_R$ higher than 0.37—a change in the stock risk parameter affects neither the investment strategy nor the insurer’s safety level. The four cases are summarised in Table 2.
Figure 3  Fraction of risky stock investments and DPO depending on $\text{shock}_{st}$

Table 2  Adjustment of the insurer’s investment strategy and default risk level to a marginal change in $\text{shock}_{st}$.

<table>
<thead>
<tr>
<th>Regulation</th>
<th>Lax</th>
<th>...</th>
<th>strict</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>I</td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>$\text{shock}_{st}$</td>
<td>$\leq 0.28$</td>
<td>$0.28 - 0.35$</td>
<td>$0.35 - 0.37$</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>-</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$DPO$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td>-</td>
</tr>
</tbody>
</table>

4.3 Influence of Insurer’s Asset-Liability Correlation

We next investigate a situation in which the insurer can change the correlation between its asset and liability risks without attracting the regulator’s attention. This could be done, e.g., by acquiring stocks of companies that suffer from risks that are also in the insurer’s liability portfolio or by investing in insurance-linked securities. We look at how the insurer will adjust its fraction of risky stock investments $\alpha^*$ in light of the new risk correlation. On the one hand, this allows us to discover how robust the standard formula is with regard to a change in the correlation. On the other hand, and even more importantly, we discover whether the
An insurer has an incentive to adjust its asset-liability strategy and take risks under a certain regulatory correlation coefficient.

Figure 4 illustrates the insurer’s optimal stock investments for three different asset-liability correlations $\rho \in \{-0.5, -0.25, 0\}$. The standard case from above ($\rho = -0.25$) is shown by the solid line. Here, high liability values are positively related to low asset values, meaning that these risks are rarely diversified. For $\rho = -0.5$ (dashed line), the coincidence of high liability and low asset values is even stronger, meaning that the asset-liability profile has become even more risky. Since the regulator cannot observe this change, capital requirements are relatively lax in this situation. Thus, the insurer decides on a higher stock investment. As the standard formula does not fully account for the risk shifting, $\rho = -0.5$ yields a higher $DPO$ and increases $SHV$ (see Figure 5). For $\rho = 0$ (dotted line), asset and liability risks are diversified, meaning that the asset-liability portfolio is less risky than in the previous cases. Since the capital requirements based on the standard formula are relatively strict now, the insurer reduces the share of risky investments, which results in a decrease of $DPO$ and $SHV$.

In short, the insurer will take advantage of opportunities that realise a positive relation between asset and liability risk, as long as the regulator cannot observe this behaviour. This allows the insurer to hold a relatively risky asset-liability portfolio without a corresponding capital add-on. The insurer’s incentive to avoid diversification between asset and liability risks is particularly high if the stock risk parameter in the standard formula is low (see Figure 5).
Figure 4  Influence of the correlation between asset and liability risks \( \rho \) on the optimal stock investments \( \alpha^* \).

Figure 5  Influence of the correlation between asset and liability risks \( \rho \) on DPO.
5 Summary and outlook

This article explores an insurer’s optimal capital and investment strategy when capital requirements are based on a standard formula. To that end, the shareholder value, which serves as the insurer’s objective function, is evaluated by option pricing techniques. Since insurance buyers do not adjust willingness to pay to insurer default risk, the insurer aims at maximizing the value of its default put option. This situation explains why insurers aim at holding a significant share of risky investments without providing additional capital, which is a problem that Solvency II is intended to remedy. Our results show that the calibration of the standard model has a strong influence on the insurer’s capital and investment strategy, and also on its default risk level. In addition to the problems of statistical errors, which have been identified by Mittnik (2011), we thus stress the importance of paying attention to the incentives that result from the formulation of capital requirements.

Our analysis shows that the influences of the standard model parameter on the insurer’s optimal strategy can be categorised into four cases. Which of these cases will manifest depends on each insurer’s individual characteristics, such as the dependency between its asset and liability risks. Therefore, it may be that the model will induce some insurers to purchase risky assets while others will invest completely risk-free. Monitoring the adjustment in insurers’ investment strategies following first implementation of the standard model, as well as parameter changes, will be a very useful way for regulators to discover whether the standard model assumptions actually fit an insurer’s characteristics.

As we mentioned in the introduction, our approach is based on the situation where the insurer’s capital level is solely defined by the regulatory requirement. However, we think that the approach could also work in the presence of market discipline, i.e., when demand provides insurers with incentives to hold more capital than the regulatory minimum. This is because insurers’ safety levels are often reported as a percentage of the regulatory requirement and insurance buyers could well be influenced by this number. Furthermore, it seems likely that the regulatory requirements under Solvency II could serve as a benchmark for the internal modeling as well as the risk measurement processes of rating agencies (Mittnik, 2011, p. 2). Investigating the side-effects of the standard model calibration on insurance demand and insurers’ optimal risk management strategy would be an important extension of the analysis conducted in this article. For example, one could determine those calibration parameters that optimise a certain regulatory objective function, such as the consumer surplus (cf. Stoyanova
et al., 2011). It would be interesting to find out whether these “optimal” calibration factors coincide with the “objective” statistical estimates.
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