A DYNAMIC PROGRAMMING ALGORITHM
FOR THE VALUATION OF GUARANTEED MINIMUM
WITHDRAWAL BENEFITS IN VARIABLE ANNUITIES
UNDER THE HESTON FRAMEWORK

ANNA RITA BACINELLO o[1], PIETRO MILLOSOVICH o[2],
AND ALVARO MONTEALEGRE o[3]

Abstract. In this paper we describe a dynamic programming algorithm for
pricing variable annuities with Guaranteed Minimum Withdrawal Benefits
(GMWB) in discrete times under the Heston framework. The GMWB promises
to return to the annuitant at least her entire initial investment, although spread
over an extended period of time. We adopt a dynamic annuitant behavior, un-
der which she decides whether to withdraw a fixed guaranteed withdrawal
amount, to surrender the contract, or to do nothing. We perform a sensitivity
analysis for different parameters of the model and compare the results with
those obtained in the Black Scholes framework.

1. Introduction

A variable annuity with Guaranteed Minimum Withdrawal Benefits (GMWB) is
a contract between a policyholder and an insurance company. At inception of the
contract, the policyholder makes a single premium payment and can choose to invest
the premium within a range of mutual funds. Thus she has the equity participation
in a portfolio of risky assets. During the life of the product the policyholder can
withdraw a specified guaranteed amount each year and the insurance company
promises to return (at least) her entire initial investment regardless of the
performance of the risky assets. The policyholder can also surrender the contract,
subject to a possible penalty. At contract expiration, she is entitled to the maximum
between the current value of her remaining investments (invested account value)
and the remaining guaranteed amount (Guaranteed withdrawals account value).
This contract allows then the policyholder to participate in market upswells, while
providing a certain minimum guaranteed cash-flow. In return for providing this
guarantee, each year the insurance company applies a proportional fee to the
invested account value.

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Statistics 'B. de Finetti' – University of Trieste, Piazzale Europa 1, 34127 Trieste, Italy.
© Department of financial economics, Castilla La Mancha University, Ciudad Real, Spain.
o[1] bacinello@units.it
o[2] pietro.millosovich@econ.units.it
o[3] alvaro.montealegre@uclm.es.