Abstract

A multiscale SABR model that describes the dynamics of forward prices/rates is introduced. The model is a system of three stochastic differential equations whose independent variable is time and whose dependent variables are the forward prices/rates variable and two stochastic volatilities varying on two different time scales. That is the multiscale SABR model couples an asset price dynamics equation with a two factors volatility model. This multiscale model generalizes the SABR model introduced in 2002 by Hagan, Kumar, Lesniewski, Woodward in [1]. The SABR model is a system of two stochastic differential equations that contains a one factor volatility model. Several empirical studies have shown that two factors volatility models describe the behaviour of financial prices more accurately than one factor volatility models. These studies motivate the introduction of multiscale SABR models. The SABR model [1] is characterized by a correlation coefficient and by two parameters: the $\beta$ volatility $\beta \in [0,1]$ and the volatility of volatility $\alpha$. The normal and lognormal SABR models correspond to the choices $\beta = 0$ and $\beta = 1$ respectively. The normal and lognormal multiscale SABR models are the straightforward generalizations to the multiscale situation respectively of the normal and of the lognormal SABR models. We study the normal and lognormal SABR and multiscale SABR models. Under some hypotheses on their correlation coefficients, “easy to use” formulae for the transition probability density functions of the normal and lognormal multiscale SABR models and for the prices of the corresponding European put and call options are deduced. The formulae for the transition probability density functions are three dimensional integrals of explicit integrands. However due to the special form of the integrands these three dimensional integrals can be evaluated using a standard quadrature rule at the computational cost of a two dimensional integral. The method used to study the multiscale SABR models can be applied to the normal and lognormal SABR models to obtain new formulae for the transition probability density functions of these models and for the corresponding prices of the European call and put options. The formulae for the transition probability density functions of the normal and lognormal SABR models are two dimensional integrals of explicit integrands that, due to the special forms of their integrands, can be evaluated using a standard quadrature rule at the computational cost of a one dimensional integral. In the normal SABR model case the new formula obtained is an elementary formula that can be used instead of formula (formula (120) of [2]) used in the mathematical finance literature that is based on the formula for the heat kernel of the Poincaré plane known as McKean formula [1], [2]. In the lognormal SABR model case the new formula obtained is a special case of a new formula that gives the transition probability density function of the Hull and White stochastic volatility model [3] in presence of (possibly nonzero) correlation between the stochastic differentials appearing in the price/rate and in the volatility equations. The formulae for the transition probability density functions derived in this paper are based on some recent results of Yakubovic [4] on the heat kernel for the Lebedev Kontorovich Transform. Starting from a set of option prices using the option pricing formulae announced above and the least squares method a calibration procedure for the normal and lognormal SABR and multiscale SABR models is developed. The calibration procedure is tested using real data. The real data studied are time series of exchange rates between currencies (euro/U.S. dollar, for short EUR/USD), and of prices of European call and put options on the Eurodollar futures price, in particular we study the prices of euro Foreign eXchange (for short FX) European-Style options. Finally option price data on the five year interest rate swap futures price are studied. Forecast option prices obtained with the previous option pricing formulae are compared with the prices actually observed. This comparison establishes the excellent quality of the forecast prices and as a consequence of the models and of the calibration procedure used in this forecasting experiment.


