This paper focuses on a dynamic investment strategies a pension plan can fit to guarantee a fix rate life retirement annuity. The risk suffered by pension provider comes out from the uncertainty of both stochastic financial returns of the investment of the residual amount (financial risk) and of the pensioners’ future lifetimes (demographic risk). There are many papers which have dealt with the financial and longevity risks in the decumulation phase of defined contribution pension schemes (see Blake et al. (2002), Gerrard et al. (2006)), while with this paper we would underline the kind of risk suffered by those who have an obligation to pay a fixed rate annuity. There are two opposing requirements in setting the constant rate of the instalment annuity: on one hand the interest of the pensioner to have the highest possible rate given the initial invested amount; on the other hand the interest of the pension provider, which has to manage the risk of having to use its own capital in order to ensure the payment of the retirement annuity. If the death of the pensioner occurs when the residual amount is still positive, there is a "gain" for the pension provider; conversely, if the residual amount hits zero-level while the pensioner is still alive, the pension provider has to release other own reserves to the payment of the pension benefits. Therefore, we refer to an “equilibrium” which could be reasonably represented by the annuity rate which makes the probability that the pension provider has to release other own reserves not exceeding a fixed level. This point of view, should be interpreted as a sort of solvency requirement for the pension provider.

From a practical point of view, we face this problem drawing inspiration from the actuarial literature on Variable Annuities (VA) (see Milevsky and Salisbury (2006)). In particular, our original contribution lies in exploit valuation models developed for Guaranteed Lifelong Withdrawal Benefit Option embedded in VA’s (Bacinello et al. (2010), Piscopo (2009), Haberman and Piscopo (2010)) and adapt them to pension schemes. We develop the model in a stochastic demographic framework and refer to Black-Scholes financial scenario.

Set a level of probability of default associated to a certain level of the guaranteed, the paper focuses on the determination of an optimal dynamic investment strategy of the surplus, taking into account the time in which a pensioner has the possibility to switch from a risk financial profile to another.

The numeric results we will propose are obtained via Monte Carlo simulations but analytical results are being implemented to enforce them.

The paper is structured as follows. At first we introduce the demographic and financial scenario in which the model operates and we describe the valuation problem. Then we present some analytical results, the simulation algorithm and finally we propose some
numerical examples.